Our Maths Curriculum will nurture enquiring and confident mathematicians who have a fluent grasp of number and mathematical concepts; appreciate the myriad of ways maths can be found in the world around them; relish opportunities to solve complex problems, finding increasingly elegant solutions for them; apply logical reasoning to challenges; and accurately communicate their mathematical thinking.

Maths: Concepts Overview

The **concepts** are the golden threads that run throughout the curriculum for each subject; they transcend context specific knowledge and skills. The concepts link directly to the N.C. subject aims.

Concept 1	Concept 2	Concept 3	Concept 4
Fluency 	**Problem Solving** 	**Reasoning** 	**Mathematical Communication** 
• **Become fluent in the fundamentals of mathematics,** including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately	• **Can solve problems by applying their mathematics** to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions	• **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language	• ***Communicate mathematical thoughts and processes, particularly to support their reasoning;*** *by using precise, mathematical vocabulary, and visual proof, with accuracy when describing, explaining, convincing, justifying and proving.* • ***To communicate their thinking clearly, verbally or in writing,*** *to themselves, as well as others.*

MATHS – Contents

Maths: Concept Milestones

The **Concept Milestones** break down the overarching concepts and indicate what pupils should achieve in each concept by the end of each Key Stage. The Milestones link directly to the N.C. subject content.

	Concept 1: Fluency	**Concept 2: Problem Solving**	**Concept 3: Reasoning**	***Concept 4: Mathematical Communication***
Milestone 1 (EYFS)	• Have a deep understanding of number to 10, including the composition of each number • Subitise (recognise quantities without counting) up to 5 • Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. • Verbally count beyond 20, recognising the pattern of the counting system • Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity • Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	• *Engage with mathematical activities and problems.* • *Independently choose to scaffold thinking using concrete and pictorial representations, if required.* • *With support (classroom discussion, paired or guided work) find a starting point to break into a problem.* • *Use trial and trial strategy.* • *Use ideas gained from a trial to decide what to do next.* • *With support find possibilities.* • *With support (adult, peer) check work (e.g. look for other possibilities and errors).* • *With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers).*	• *Describe what they have done* • *Represent thinking using concrete, pictorial or abstract representations, as appropriate.*	• *Pupils should verbally use mathematical vocabulary, at a level consistent with their increasing word knowledge at Early Years* • *Evidence their mathematical thinking verbally* • *Listen to others' descriptions.*
Milestone 2 (Yr 1/2)	• Develop confidence and mental fluency with whole numbers, counting and place value; working with numerals, words and the four operations, including with practical resources • Develop their ability to recognise, describe, draw, compare and sort	• *Make links and move between different representations (concrete, pictorial, abstract).* • *Independently choose to scaffold thinking using concrete, pictorial **or abstract representations**, if required.*	• *Explain what they have done, including some reasons for what they did (i.e. beginning to use inductive reasoning)* • *Make some use of visuals (e.g. diagrams, picture, annotations)*	• Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1. • *Evidence their mathematical thinking through verbal*

	different shapes and use the related vocabulary. • Use a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. • Know the number bonds to 20 and be precise in using and understanding place value. • An emphasis on practice at this early stage will aid fluency.	• *Independently find a starting point to break into a problem.* • *Use trial and **improvement** strategy and with support work systematically.* • ***Independently** find possibilities.* • *Independently check work (e.g. look for other possibilities, repeats, missing answers and errors).* • *Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial).* • *With support, investigate statements and conjectures.*		*statements and begin to use written evidence* • *Listen to others' explanations, make sense of them and compare and evaluate.* • *Begin to edit and improve their own and a peer's explanation.*
Milestone 3 (Yr 3/4)	• Become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. • Develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. • Develop their ability to solve a range of problems, including with simple fractions and decimal place value. • Draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. • Use measuring instruments with accuracy and make connections between measure and number. • Know the multiplication tables up to and including the 12 multiplication table • Show precision and fluency in their work.	• *Make suggestions of ways to solve a range of problems.* • *Develop and apply a systematic approach.* • *Find and predict possibilities that match the context using patterns spotted to support.* • *Independently check **and improve** work (e.g. look for other possibilities, repeats, missing answers, errors and **ways to improve**).* • *Begin to connect different but related patterns and use these to help solve problems.* • *Make and investigate conjectures and provide examples and counter-examples.* • *When they have solved a problem, pose a similar problem for a peer.*	• *Provide a convinced argument with confidence and a chain of reasoning that makes sense to themselves (i.e. Using inductive reasoning)* • *Make use of a range of mathematical visuals (including tables to organise their process/findings/thoughts)*	• Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling. • *Evidence their mathematical thinking through clear verbal statements and written sentences* • *Edit and improve their own and a peer's convinced explanation.*

Milestone 4 (Yr 5/6)	• Extend their understanding of the number system and place value to include larger integers. • Develop the connections between multiplication and division with fractions, decimals, percentages and ratio. • Develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. • Begin to use the language of algebra as a means for solving a variety of problems. • Extend knowledge developed in number through the use of geometry and measures. • Classify shapes with increasingly complex geometric properties and learn the vocabulary needed to describe them. • Be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.	• *Organise work from the outset, looking for ways to record and work systematically.* • *Identify and connect different but related patterns and use these to help solve problems.*	• *Provide a clear, correct, logical justification with a complete chain of mathematically credible reasoning - Year 5* • *Provide proof of reasoning, using a watertight argument that is mathematically sound, based on generalisations and underlying mathematical structure (i.e. deductive reasoning) – Year 6* • *Express generalisations in words and symbolic notation, including through the use of algebra* • *Make accurate use of deliberately chosen mathematical visuals (e.g. diagrams, tables, graphs)*	• Pupils should read, spell and pronounce mathematical vocabulary correctly. • *Be able to communicate their reasoning in writing without any verbal explanation/interpretation* • *Reflect on others' justification/ proof and use this to improve their own work* • *Edit and improve their own and a peer's justification – Year 5* • *Edit and improve their own and a peer's proof – Year 6* • *To ask questions of other children to help move their thinking forwards*

MATHS Learning
Nursery

Concept	Milestone	Learning
Concept 1: Fluency	1. Have a deep understanding of number to 10, including the composition of each number 2. Subitise (recognise quantities without counting) up to 5 3. Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. 4. Verbally count beyond 20, recognising the pattern of the counting system 5. Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity 6. Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	**Counting & Cardinality** a) Through play and exploration, beginning to learn that numbers are made up (composed) of smaller number b) Separates a group of three or four objects in different ways, beginning to recognise that the total is still the same c) Enjoy counting verbally as far as they can go (beyond 5) d) Uses some number names and number language within play, and may show fascination with large numbers e) Points or touches each item, saying one number for each item, using the stable order of 1,2,3,4,5 f) Counts up to five items, recognising that the last number said represents the total counted so far (cardinal principle) g) Links numerals with amounts up to 5 and maybe beyond h) Begin to recognise numerals 0 to 10
		Subitising i) Subitises one, two and three objects (without counting)
		Comparison j) Compares two small groups of up to five objects, saying when there are the same number of objects in each group k) Beginning to recognise that each counting number is one more than the one before
		Measurement l) Compare quantities using language: 'more than', 'fewer than' m) Make comparisons between objects relating to size, length, weight and capacity.
		Geometry – properties of shapes n) Chooses items based on their shape which are appropriate for the their purpose o) Responds to both informal language and common shape names p) Shows awareness of shape similarities and differences between objects q) Talk about and explore 2D and 3D shapes (for example, circles, rectangles, triangles and cuboids) using informal and mathematical language: 'sides', 'corners'; 'straight', 'flat', 'round'. r) Enjoys partitioning and combining shapes to make new shapes with 2D and 3D shapes s) Attempts to create arches and enclosures when building, using trial and improvement to select blocks
		Geometry – position and direction t) Responds to and uses language of position and direction, using words like 'in front' or 'behind' u) Predicts, moves and rotates objects to fit the space or create the shape they would like v) Describes a familiar route
Concept 2: Problem Solving	1. *Engage with mathematical activities and problems.* 2. *Independently choose to scaffold thinking using*	**Engage with mathematical activities and problems.** a) Be curious about mathematical problems and be willing to have a go b) Solve real world mathematical problems with numbers up to 5
		Independently choose to scaffold thinking using concrete and pictorial representations, if required

Concept	Milestone	Learning
	concrete and pictorial representations, if required. *3. With support (classroom discussion, paired or guided work) find a starting point to break into a problem.* *4. Use trial and trial strategy.* *5. Use ideas gained from a trial to decide what to do next.* *6. With support find possibilities.* *7. With support (adult, peer) check work (e.g. look for other possibilities and errors).* 8. *With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers).*	c) Know that physical objects and pictures can help to break down thinking when solving a problem d) Begin to use their chosen representations to explore the problem and seek solutions
		With support (classroom discussion, paired or guided work) find a starting point to break into a problem. *e)* Use a variety of everyday manipulatives to explore problems f) b) Begin to choose concrete and pictorial representations to approach and solve problems
		Use trial and trial strategy. g) Use a trial and trial approach when trying to solve a problem *h)* Make a simple suggestion about how to solve the problem based on prior knowledge and experiences **Use ideas gained from a trial to decide what to do next.** i) Following initial trials and what they have found out, make a decision about what to do next j) Reflect on the best approach
		With support find possibilities. k) Begin to understand that there may be more than one way to solve a problem l) With support, find a different solution m) Explore simple problems where there can be more than one answer n) Begin to know that there can be more than one answer/way to solve a problem *o)* Work alongside an adult or peer to find more than one answer
		With support (adult, peer) check work (e.g. look for other possibilities and errors). *p)* To begin to notice where there may be an error in a problem *q)* With support, develop and use simple strategies to check r) Begin to understand why checking is important
		With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers). *s) Talk about and identify the patterns around them (fabrics, everyday objects and materials, movement and music, events)* *t) Spots patterns in the environment, beginning to identify the pattern "rule"* *u) Create their own spatial patterns showing some organisation or regularity* *v) Extend and create patterns with varying rules (including AB, ABB, ABBC)* *w) Notice and correct an error in a repeating pattern* x) *f) Begin to describe a sequence of events, real of fictional, using words such as 'first', 'then'…*
Concept 3: Reasoning	1. *Describe what they have done* 2. *Represent thinking using concrete, pictorial or abstract representations, as appropriate.*	***Describe what they have done*** *a) Talk about what they have noticed and found out* *b) Explain why they think something and/or how they know* c) *Begin to make links with what they already know (e.g. have you seen something like this before?)*
		Represent thinking using concrete, pictorial or abstract representations, as appropriate. *d) Use concrete objects to represent their thinking* *e) Begin to use drawings, sketches and labels to represent their thinking* *f) Begin to us mathematical symbols such as '+' and '-' (where appropriate) to represent their thinking*

Concept	Milestone	Learning
Concept 4: Mathematical Communication	1. *Pupils should verbally use mathematical vocabulary, at a level consistent with their increasing word knowledge at Early Years* 2. *Evidence their mathematical thinking verbally* 3. *Listen to others' descriptions.*	a) *To use a combination of everyday and age appropriate mathematical vocabulary to talk about and explain their thinking and ideas.* b) *To verbally explain the steps they took to solve a problem, so that their thinking/process is either understood or can be inferred.* c) *To begin to give some simple reasons for their thinking and explain how they know.* d) *To listen and respond to others' ideas (e.g. saying whether they agree or disagree)* e) *To begin to respond to others' ideas by comparing it to their own thinking*

MATHS Learning
Reception

Concept	Milestone	Learning
Concept 1: Fluency	1. Have a deep understanding of number to 10, including the composition of each number 2. Subitise (recognise quantities without counting) up to 5 3. Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts. 4. Verbally count beyond 20, recognising the pattern of the counting system 5. Compare quantities up to 10 in different contexts, recognising when one quantity is greater than, less than or the same as the other quantity 6. Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	**Counting & Cardinality** a) Count objects, actions and sounds b) *Knows that when objects are moved the total doesn't change* c) Enjoys reciting numbers from 0-10 (and beyond) and back from 10 to 0 d) Uses number names and symbols when comparing numbers, showing interest in large numbers e) Match the numeral with a group of items to sow how many there are (up to 10) f) Link the number symbol (numeral) with its cardinal number value g) Counts out up to 10 objects from a larger group h) Count beyond 10 i) Be increasingly confident at putting numerals in order 0 to 10 (ordinality)
		Composition j) Explore the composition of numbers to 10 k) Show awareness that numbers are made up (composed) of smaller numbers, exploring partitioning in different ways with a wide range of objects Automatically recall number bonds for numbers 0–5 and some to 10
		Subitising l) Engages in subitising numbers Begin to conceptually subitise larger numbers by subitising smaller groups within a number
		Comparision m) Compare numbers n) Estimate numbers of things, showing understanding of relative size o) Understand the 'one more than/one less than' relationship between consecutive numbers p) Distribute items evenly, sharing items/objects equally q) Understand that numbers that can't be shared equally are odd, and those that can are even In practical activities, add one and subtract one with numbers to 10
		Measurement r) Explore measures using every day objects and mathematical apparatus s) Use language relating to size, weight and capacity to compare measures *Begin to use time to sequence events*
		Geometry – properties of shape t) Uses informal language and analogies, (e.g. heart shaped and hand-shaped leaves), as well as mathematical terms to describe shapes u) Talk about and explore 2D and 3D shapes (for example, circles, rectangles, triangles and cuboids) using informal and increasing mathematical language: 'sides', 'corners'; 'straight', 'flat', 'round'

Concept	Milestone	Learning
		v) Uses own ideas to make models of increasing complexity, selecting blocks needed, solving problems and visualising what they will build Enjoys composing and decomposing shapes, learning which shapes combine to make other shapes
		Geometry – position and direction w) Uses spatial language, including following and giving directions, using relative terms and describing what they see from different viewpoints
Concept 2: Problem Solving	1. *Engage with mathematical activities and problems.* 2. *Independently choose to scaffold thinking using concrete and pictorial representations, if required.* 3. *With support (classroom discussion, paired or guided work) find a starting point to break into a problem.* 4. *Use trial and trial strategy.* 5. *Use ideas gained from a trial to decide what to do next.* 6. *With support find possibilities.* 7. *With support (adult, peer) check work (e.g. look for other possibilities and errors).* 8. *With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers).*	**Engage with mathematical activities and problems.** a) Be curious about mathematical problems and be willing to have a go b) Solve real world mathematical problems with numbers up to 5
		Independently choose to scaffold thinking using concrete and pictorial representations, if required c) Know that physical objects and pictures can help to break down thinking when solving a problem d) Begin to use their chosen representations to explore the problem and seek solutions
		With support (classroom discussion, paired or guided work) find a starting point to break into a problem. *e)* Use a variety of everyday manipulatives to explore problems *f)* Begin to choose concrete and pictorial representations to approach and solve problems
		Use trial and trial strategy. g) Use a trial and trial approach when trying to solve a problem *h)* Make a simple suggestion about how to solve the problem based on prior knowledge and experiences **Use ideas gained from a trial to decide what to do next.** i) Following initial trials and what they have found out, make a decision about what to do next j) Reflect on the best approach
		With support find possibilities. k) Begin to understand that there may be more than one way to solve a problem l) With support, find a different solution m) Explore simple problems where there can be more than one answer n) Begin to know that there can be more than one answer/way to solve a problem *o)* Work alongside an adult or peer to find more than one answer
		With support (adult, peer) check work (e.g. look for other possibilities and errors). *p)* To begin to notice where there may be an error in a problem *q)* With support, develop and use simple strategies to check *r)* Begin to understand why checking is important
		With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers). *s) Talk about and identify the patterns around them (fabrics, everyday objects and materials, movement and music, events)* *t) Spots patterns in the environment, beginning to identify the pattern "rule"* *u) Create their own spatial patterns showing some organisation or regularity*

Concept	Milestone	Learning
		v) Extend and create patterns with varying rules (including AB, ABB, ABBC) *w) Notice and correct an error in a repeating pattern* *x) Begin to describe a sequence of events, real of fictional, using words such as 'first', 'then'…*
Concept 3: Reasoning	*1. Describe what they have done* *2. Represent thinking using concrete, pictorial or abstract representations, as appropriate.*	***Describe what they have done*** *a) Talk about what they have noticed and found out* *b) Explain why they think something and/or how they know* c) *Begin to make links with what they already know (e.g. have you seen something like this before?)*
		Represent thinking using concrete, pictorial or abstract representations, as appropriate. *d) Use concrete objects to represent their thinking* *e) Begin to use drawings, sketches and labels to represent their thinking* f) *Begin to us mathematical symbols such as '+' and '-' (where appropriate) to represent their thinking*
Concept 4: Mathematical Communication	*1. Pupils should verbally use mathematical vocabulary, at a level consistent with their increasing word knowledge at Early Years* *2. Evidence their mathematical thinking verbally* 3. *Listen to others' descriptions.*	a) *To use a combination of everyday and age appropriate mathematical vocabulary to talk about and explain their thinking and ideas.* b) *To verbally explain the steps they took to solve a problem, so that their thinking/process is either understood or can be inferred.* c) *To begin to give some simple reasons for their thinking and explain how they know.* d) *To listen and respond to others' ideas (e.g. saying whether they agree or disagree)* e) *To begin to respond to others' ideas by comparing it to their own thinking*

MATHS Learning
Year 1

Concept	Milestone	Learning
Concept 1: Fluency	1. Develop confidence and mental fluency with whole numbers, counting and place value; working with numerals, words and the four operations, including with practical resources 2. Develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. 3. Use a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. 4. Know the number bonds to 20 and be precise in using and understanding place value. 5. An emphasis on practice at this early stage will aid fluency.	**Number & Place value** a) count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number b) count, read and write numbers to 100 in numerals; count in multiples of twos, fives and tens c) given a number, identify one more and one less d) identify and represent numbers using objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least e) read and write numbers from 1 to 20 in numerals and words.
		Addition & Subtraction f) read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs g) represent and use number bonds and related subtraction facts within 20 h) add and subtract one-digit and two-digit numbers to 20, including zero i) solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ☐ – 9.
		Multiplication & division j) solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.
		Fractions k) recognise, find and name a half as one of two equal parts of an object, shape or quantity l) recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.
		Measurement m) compare, describe and solve practical problems for: 1. lengths and heights [for example, long/short, longer/shorter, tall/short, double/half] 2. mass/weight [for example, heavy/light, heavier than, lighter than] 3. capacity and volume [for example, full/empty, more than, less than, half, half full, quarter] 4. time [for example, quicker, slower, earlier, later] n) measure and begin to record the following: 1. lengths and heights 2. mass/weight 3. capacity and volume 4. time (hours, minutes, seconds) o) recognise and know the value of different denominations of coins and notes p) sequence events in chronological order using language [for example, before and after, next, first, today, yesterday, tomorrow, morning, afternoon and evening] q) recognise and use language relating to dates, including days of the week, weeks, months and years

Concept	Milestone	Learning
		r) tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.
		Geometry – properties of shapes s) recognise and name common 2-D and 3-D shapes, including: 1. 2-D shapes [for example, rectangles (including squares), circles and triangles] 2. 3-D shapes [for example, cuboids (including cubes), pyramids and spheres].
		Geometry – position & direction t) describe position, direction and movement, including whole, half, quarter and three-quarter turns.
Concept 2: Problem Solving	*1. Make links and move between different representations (concrete, pictorial, abstract).* *2. Independently choose to scaffold thinking using concrete, pictorial **or abstract representations**, if required.* *3. Independently find a starting point to break into a problem.* *4. Use trial and **improvement** strategy and with support work systematically.* *5. **Independently** find possibilities.* *6. Independently check work (e.g. look for other possibilities, repeats, missing answers and errors).* *7. Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial).* 8. *With support, investigate statements and conjectures.*	***Make links and move between different representations (concrete, pictorial, abstract).*** *a) Know what concrete representations (*numicon*, base ten, counters,* unifix*) are and how they can be used to represent a problem* *b) Use/select concrete representations (*numicon*, base ten, counters,* unifix*) to explore problems* *c) Use pictorial representations (bar model, part-whole, place value grid, 10 frame)* *d) Create pictorial representations (bar model, part-whole, place value grid, 10 frame) to solve a problem* *e) Identify and comment on similarities, differences and relationships between different representations (concrete, pictorial, abstract)* *f) Use abstract representations (2+4=6) alongside concrete and pictorial representations and make links*
		Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required. *g) Know how concrete, (numicon, base ten etc.) pictorial (e.g. bar models, part-whole, number lines, place value charts) and abstract representations can be used to break down thinking into smaller chunks when solving a problem* h) *Use chosen representations to effectively make their thinking visual and help solve problems*
		Independently find a starting point to break into a problem. *i) Draw diagrams to understand a problem (array/pictures/concept cartoon) collaboratively or with support initially then independently* *j) Represent what a problem is asking with a bar model, part-whole model* *k) Act out or orally tell a story to understand what a problem is (e.g. I had 4 apples and ate 2)* *l) Talk to a partner about a problem and ask clarifying questions (I think we need to...Do you think we should...Where should we start? Did you do this first because?)* *m) Identify maths skills that will be needed to solve a problem (from a supporting list initially with e.g. addition, place value, measure)* *n) Recall previous experiences of similar problems and use them to approach new learning (When we solved this problem...This is similar to when...In real life I have...)*
		Use trial and improvement strategy and with support work systematically. *o) Know what trial and improvement is as a strategy for problem solving* *p) Make appropriate guesses or estimations about a problem* *q) Choose a logical first and next step to solve a problem* *r)* Consider what an initial guess reveals about the problem and use this to attempt the problem again *s)* Begin to use and understand the language of systematically (as logical, organised)

Concept	Milestone	Learning
		t) Give reasons for a starting point and make an improvement *u)* Adapt and improve working out to get closer to an answer *v)* *Reflect on guesses and estimations about a problem with a peer or adult*
		Independently find possibilities. *w) Begin to consider how you might get all possibilities within a problem (Have you got them all? How do you know?)* *x) Find some solutions to a problem* *y) Explore problems where there can be more than one possibility* *z) Know that there can be more than one possible answer to a problem* *aa) Work alongside a peer to find more than one possible answer* *bb) Sort possibilities to determine which ones could be missing* *cc) Choose strategies and use prior mathematical knowledge and experiences to find more than one answer (e.g. the inverse, related number facts) or know there can be more than one viable possibility* *dd) Recognise that some answers to a problem are the same* *ee) Recognise how some answers are different*
		Independently check learning (e.g. look for other possibilities, repeats, missing answers and errors). *ff) Spot errors in a problem worked on collaboratively* *gg) Develop strategies for checking (working backwards, comparing with a partner, re-attempting the problem, explaining solutions aloud to a peer or adult, check-list, use a different representation (e.g. concrete instead of pictorial)* *hh) Know what an error could look like in a problem* *ii) Re-visit a problem and check they have met success criteria* *jj) Understand what checking is and why it is important* *kk) Notice when there are other possibilities or missing answers in their own or others' solutions* *ll) Understand why repeats can not be included in a solution*
		Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial). *mm) Know what patterns and sequences are and what they can look like in Maths* *nn) Notice and identify simple number/shape/spatial patterns* *oo) Predict what could come next in a simple pattern or sequence* *pp) Continue a simple pattern or sequence created by a peer*
		With support, investigate statements and conjectures. *qq) Collaboratively explore statements including: always, sometimes, never; true and false; the odd one out is...because...If...then.. If not...then...If only...then* *rr) Collaboratively explore opinions formed in the process/as part of solving a problem (I think...they think...we should then...)*
Concept 3: Reasoning	*1. Explain what they have done, including some reasons for what they did (i.e. beginning to use inductive reasoning)*	*a) Understand that problem solving and reasoning are linked.* *b) Understand that they are using reasoning skills when solving problems* *c) To explain how what they have done links to the problem they are solving.*

Concept	Milestone	Learning
	2. Make some use of visuals (e.g. diagrams, picture, annotations)	***Explain what they have done, including some reasons for what they did (i.e. beginning to use inductive reasoning)*** *d) To offer some reasons for what they have done (which may or may not be mathematically correct)* e) *To begin to provide an explanation for what they did and begin to connect the reasons (although these may not hang together coherently yet)*
		Make some use of visuals (e.g. diagrams, picture, annotations) *f) To use diagrams and/or visuals that have a mathematical relevance and purpose to demonstrate their understanding* *g) To annotate the visuals with mathematically relevant (age appropriate) vocabulary* h) *To organise and collate their visuals, notes and calculations so they have some semblance of organisation and structure, and relate to their thought process.*
Concept 4: Mathematical Communication	1. Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1. *2. Evidence their mathematical thinking through verbal statements and begin to use written evidence* *3. Listen to others' explanations, make sense of them and compare and evaluate.* 4. *Begin to edit and improve their own and a peer's explanation.*	***Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.*** a) *To use mathematical vocabulary, with increasing regularity to explain their thinking and ideas.*
		Evidence their mathematical thinking through verbal statements and begin to use written evidence *b) To verbally explain the steps they took to solve a problem, so that their thinking/process is clearly understood.* *c) To give reasons for their thinking and explain how they know.* *d) To write simple sentences based on sentence stems.*
		Listen to others' explanations, make sense of them and compare and evaluate. *e) To ask clarification questions while listening to other children's explanations* *f) To respond to others' explanations by comparing it to their own thinking* *g) To comment on the effectiveness of others' explanations*
		Begin to edit and improve their own and a peer's explanation. *h) To improve their explanations based on feedback.* *i) To begin to suggest how others might improve their explanations*

MATHS Learning
Year 2

Concept	Milestone	Learning
Concept 1: Fluency	a) Develop confidence and mental fluency with whole numbers, counting and place value; working with numerals, words and the four operations, including with practical resources b) Develop their ability to recognise, describe, draw, compare and sort different shapes and use the related vocabulary. c) Use a range of measures to describe and compare different quantities such as length, mass, capacity/volume, time and money. d) Know the number bonds to 20 and be precise in using and understanding place value. e) An emphasis on practice at this early stage will aid fluency.	**Number & place value** a) count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward b) recognise the place value of each digit in a two-digit number (tens, ones) c) identify, represent and estimate numbers using different representations, including the number line d) compare and order numbers from 0 up to 100; use <, > and = signs e) read and write numbers to at least 100 in numerals and in words f) use place value and number facts to solve problems.
		Addition & subtraction g) solve problems with addition and subtraction: 1. using concrete objects and pictorial representations, including those involving numbers, quantities and measures 2. applying their increasing knowledge of mental and written methods h) recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 i) add and subtract numbers using concrete objects, pictorial representations, and mentally, including: 1. a two-digit number and ones 2. a two-digit number and tens 3. two two-digit numbers 4. adding three one-digit numbers j) show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot k) recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.
		Multiplication & division l) recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers m) calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs n) show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot o) solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.
		Fractions p) recognise, find, name and write fractions $\frac{1}{3}$, $\frac{1}{4}$, $\frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity q) write simple fractions for example, $\frac{1}{2}$ of 6 = 3 and recognise the equivalence of $\frac{2}{4}$ and $\frac{1}{2}$.

Concept	Milestone	Learning
		Measurement r) choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature (°C); capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels s) compare and order lengths, mass, volume/capacity and record the results using >, < and = t) recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value u) find different combinations of coins that equal the same amounts of money v) solve simple problems in a practical context involving addition and subtraction of money of the same unit, including giving change w) compare and sequence intervals of time x) tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times y) know the number of minutes in an hour and the number of hours in a day.
		Geometry – properties of shapes z) identify and describe the properties of 2-D shapes, including the number of sides and line symmetry in a vertical line aa) identify and describe the properties of 3-D shapes, including the number of edges, vertices and faces bb) identify 2-D shapes on the surface of 3-D shapes [for example, a circle on a cylinder and a triangle on a pyramid] cc) compare and sort common 2-D and 3-D shapes and everyday objects.
		Geometry – position & direction dd) order and arrange combinations of mathematical objects in patterns and sequences ee) use mathematical vocabulary to describe position, direction and movement, including movement in a straight line and distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise).
		Statistics ff) interpret and construct simple pictograms, tally charts, block diagrams and simple tables gg) ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity hh) ask and answer questions about totalling and comparing categorical data.
Concept 2: Problem Solving	1. *Make links and move between different representations (concrete, pictorial, abstract).* 2. *Independently choose to scaffold thinking using concrete, pictorial* ***or abstract representations****, if required.* 3. *Independently find a starting point to break into a problem.*	***Make links and move between different representations (concrete, pictorial, abstract).*** *a) Know what concrete representations (*numicon*, base ten, counters,* unifix*) are and how they can be used to represent a problem* *b) Use/select concrete representations (*numicon*, base ten, counters,* unifix*) to explore problems* *c) Use pictorial representations (bar model, part-whole, place value grid, 10 frame)* *d) Create pictorial representations (bar model, part-whole, place value grid, 10 frame) to solve a problem* *e) Identify and comment on similarities, differences and relationships between different representations (concrete, pictorial, abstract)* *f) Use abstract representations (2+4=6) alongside concrete and pictorial representations and make links*

Concept	Milestone	Learning
	4. *Use trial and **improvement** strategy and with support work systematically.* 5. ***Independently** find possibilities.* 6. *Independently check work (e.g. look for other possibilities, repeats, missing answers and errors).* 7. *Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial).* 8. *With support, investigate statements and conjectures.*	***Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.*** *g) Know how concrete, (numicon, base ten etc.) pictorial (e.g. bar models, part-whole, number lines, place value charts) and abstract representations can be used to break down thinking into smaller chunks when solving a problem* *h) Use chosen representations to effectively make their thinking visual and help solve problems*
		Independently find a starting point to break into a problem. *i) Draw diagrams to understand a problem (array/pictures/concept cartoon) collaboratively or with support initially then independently* *j) Represent what a problem is asking with a bar model, part-whole model* *k) Act out or orally tell a story to understand what a problem is (e.g. I had 4 apples and ate 2)* *l) Talk to a partner about a problem and ask clarifying questions (I think we need to...Do you think we should...Where should we start? Did you do this first because?)* *m) Identify maths skills that will be needed to solve a problem (from a supporting list initially with e.g. addition, place value, measure)* *n) Recall previous experiences of similar problems and use them to approach new learning (When we solved this problem...This is similar to when...In real life I have...)*
		Use trial and improvement strategy and with support work systematically. *o) Know what trial and improvement is as a strategy for problem solving* *p) Make appropriate guesses or estimations about a problem* *q) Choose a logical first and next step to solve a problem* *r)* Consider what an initial guess reveals about the problem and use this to attempt the problem again *s)* Begin to use and understand the language of systematically (as logical, organised) *t)* Give reasons for a starting point and make an improvement *u)* Adapt and improve working out to get closer to an answer *v) Reflect on guesses and estimations about a problem with a peer or adult*
		Independently find possibilities. *w) Begin to consider how you might get all possibilities within a problem (Have you got them all? How do you know?)* *x) Find some solutions to a problem* *y) Explore problems where there can be more than one possibility* *z) Know that there can be more than one possible answer to a problem* *aa) Work alongside a peer to find more than one possible answer* *bb) Sort possibilities to determine which ones could be missing* *cc) Choose strategies and use prior mathematical knowledge and experiences to find more than one answer (e.g. the inverse, related number facts) or know there can be more than one viable possibility* *dd) Recognise that some answers to a problem are the same* *ee) Recognise how some answers are different*

Concept	Milestone	Learning
		Independently check learning (e.g. look for other possibilities, repeats, missing answers and errors). *ff) Spot errors in a problem worked on collaboratively* *gg) Develop strategies for checking (working backwards, comparing with a partner, re-attempting the problem, explaining solutions aloud to a peer or adult, check-list, use a different representation (e.g. concrete instead of pictorial)* *hh) Know what an error could look like in a problem* *ii) Re-visit a problem and check they have met success criteria* *jj) Understand what checking is and why it is important* *kk) Notice when there are other possibilities or missing answers in their own or others' solutions* *ll) Understand why repeats can not be included in a solution*
		Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial). *mm) Know what patterns and sequences are and what they can look like in Maths* *nn) Notice and identify simple number/shape/spatial patterns* *oo) Predict what could come next in a simple pattern or sequence* *pp) Continue a simple pattern or sequence created by a peer*
		With support, investigate statements and conjectures. *qq) Collaboratively explore statements including: always, sometimes, never; true and false; the odd one out is...because...If...then.. If not...then...If only...then* *rr) Collaboratively explore opinions formed in the process/as part of solving a problem (I think...they think...we should then...)*
Concept 3: Reasoning	*1. Explain what they have done, including some reasons for what they did (i.e. beginning to use inductive reasoning)* *2. Make some use of visuals (e.g. diagrams, picture, annotations)*	*a) Understand that problem solving and reasoning are linked.* *b) Understand that they are using reasoning skills when solving problems* *c) To explain how what they have done links to the problem they are solving.*
		Explain what they have done, including some reasons for what they did (i.e. beginning to use inductive reasoning) *d) To offer some reasons for what they have done (which may or may not be mathematically correct)* *e) To begin to provide an explanation for what they did and begin to connect the reasons (although these may not hang together coherently yet)*
		Make some use of visuals (e.g. diagrams, picture, annotations) *f) To use diagrams and/or visuals that have a mathematical relevance and purpose to demonstrate their understanding* *g) To annotate the visuals with mathematically relevant (age appropriate) vocabulary* *h) To organise and collate their visuals, notes and calculations so they have some semblance of organisation and structure, and relate to their thought process.*
Concept 4: Mathematical Communication	1. Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.	***Pupils should read and spell mathematical vocabulary, at a level consistent with their increasing word reading and spelling knowledge at key stage 1.*** a) *To use mathematical vocabulary, with increasing regularity to explain their thinking and ideas.*
		Evidence their mathematical thinking through verbal statements and begin to use written evidence

Concept	Milestone	Learning
	2. *Evidence their mathematical thinking through verbal statements and begin to use written evidence* 3. *Listen to others' explanations, make sense of them and compare and evaluate.* 4. *Begin to edit and improve their own and a peer's explanation.*	*b) To verbally explain the steps they took to solve a problem, so that their thinking/process is clearly understood.* *c) To give reasons for their thinking and explain how they know* *d) To write simple sentences based on sentence stems.* ***Listen to others' explanations, make sense of them and compare and evaluate.*** *e) To ask clarification questions while listening to other children's explanations* *f) To respond to others' explanations by comparing it to their own thinking* *g) To comment on the effectiveness of others' explanations* ***Begin to edit and improve their own and a peer's explanation.*** *h) To improve their explanations based on feedback* *i) To begin to suggest how others might improve their explanations*

MATHS Learning
Year 3

Concept	Milestone	Learning
Concept 1: Fluency	1. Become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. 2. Develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. 3. Develop their ability to solve a range of problems, including with simple fractions and decimal place value. 4. Draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. 5. Use measuring instruments with accuracy and make connections between measure and number. 6. Know the multiplication tables up to and including the 12 multiplication table 7. Show precision and fluency in their work.	**Number & place value** a) count from 0 in multiples of 4, 8, 50 and 100; find 10 or 100 more or less than a given number b) recognise the place value of each digit in a three-digit number (hundreds, tens, ones) c) compare and order numbers up to 1000 d) identify, represent and estimate numbers using different representations e) read and write numbers up to 1000 in numerals and in words f) solve number problems and practical problems involving these ideas.
		Addition & subtraction 1. add and subtract numbers mentally, including: • a three-digit number and ones • a three-digit number and tens • a three-digit number and hundreds 2. add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction 3. estimate the answer to a calculation and use inverse operations to check answers 4. solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.
		Multiplication & division 5. recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables 6. write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods 7. solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.
		Fractions 8. count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10 9. recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators 10. recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators 11. recognise and show, using diagrams, equivalent fractions with small denominators 12. add and subtract fractions with the same denominator within one whole [for example, $\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$] 13. compare and order unit fractions, and fractions with the same denominators 14. solve problems that involve all of the above.

Concept	Milestone	Learning
		Measurement 15. measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml) 16. measure the perimeter of simple 2-D shapes 17. add and subtract amounts of money to give change, using both £ and p in practical contexts 18. tell and write the time from an analogue clock, including using Roman numerals from I to XII, and 12-hour and 24-hour clocks 19. estimate and read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes and hours; use vocabulary such as o'clock, a.m./p.m., morning, afternoon, noon and midnight 20. know the number of seconds in a minute and the number of days in each month, year and leap year 21. compare durations of events [for example to calculate the time taken by particular events or tasks].
		Geometry – properties of shapes 22. draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations and describe them 23. recognise angles as a property of shape or a description of a turn 24. identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle 25. identify horizontal and vertical lines and pairs of perpendicular and parallel lines.
		Statistics 26. interpret and present data using bar charts, pictograms and tables 27. solve one-step and two-step questions [for example, 'How many more?' and 'How many fewer?'] using information presented in scaled bar charts and pictograms and tables.
Concept 2: Problem Solving	1. *Make suggestions of ways to solve a range of problems.* 2. *Develop and apply a systematic approach.* 3. *Find and predict possibilities that match the context using patterns spotted to support.* 4. *Independently check **and improve** work (e.g. look for other possibilities, repeats, missing answers, errors and **ways to improve**).* 5. *Begin to connect different but related patterns and use these to help solve problems.*	***Make suggestions of ways to solve a range of problems.*** a) Know and be able to identify the range/type of problems that can be solved (word problems, visual problems, finding all possibilities, logic, rules and patterns) b) Be able to identify the mathematical skill needed to solve the different types of problems (e.g. inverse, addition, multi-skill) c) Know (and use) the stages of problem solving (Nrich: Getting started, working on the problem, Digging Deeper, Reflecting) *d)* d.Know the different ways that a problem can attempted to be solved (Nrich: Trial and Improvement, pattern spotting, working backwards, visualising) e) Choose a way to solve a problem from an agreed selection f) Using prior and current mathematical knowledge or experiences to make independent suggestions to solve a problem (I could…This is where I should start because…I have started this way because...If I do this first...)
		Develop and apply a systematic approach g) Know what it means to apply a systematic approach and the skills needed h) Know how a systematic approach ensures no possibilities are left out i) Use a systematic approach to ensure no possibilities are left out j) Know and explore how first guesses can be used to solve a problem k) Decide how to adapt an initial attempt or guess at a problem to try again l) Use information from initial thoughts and guesses to plan next steps

Concept	Milestone	Learning
	6. Make and investigate conjectures and provide examples and counter- examples. *7. When they have solved a problem, pose a similar problem for a peer.*	m) Identify which information from an initial guess is useful (even if it is wrong) n) Plan steps taken to solve a problem o) *Make thought-out guesses about a problem and use these to inform next steps* p) *Refine or improve initial guesses and estimation*
		Find and predict possibilities that match the context using patterns spotted to support. q) Notice similarities and differences between examples r) Know what a mathematical pattern is s) Spot and describe patterns within a problem t) Recognise connections between possibilities and use these to find more u) Make predictions that relate to the context of a problem (e.g. I think the next number will be an odd number because 3 and 7 are both odd...) v) Generate new examples and solutions that fit the criteria of a problem based on patterns (e.g. the next numbers in the sequence could be 11 or 9)
		Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve). *w) Revisit solutions using learnt checking strategies* *x) Know what improvements can look like when solving a problem (adding in a missing number, omitting a repeat, spotting a better pattern or approach)* *y) Understand why improvements are important when problem solving* *z) Identify opportunities for improvements in their own and others' workings* *aa) Make improvements to workings that move them closer to a solution* *bb) Confirm they have found a correct solution by checking problems another way (working backwards, comparing with a partner, re-attempting the problem, explaining solutions aloud to a peer or adult, check-list, use a different representation (e.g. concrete instead of pictorial))*
		Begin to connect different but related patterns and use these to help solve problems. *cc) Understand what connect means in a mathematical context* *dd) Demonstrate an awareness of how two (or more) patterns can be related* *ee) Begin to identify similarities and differences between two (or more patterns)* *ff) Use understanding of connected patterns (e.g....) to move closer to solutions*
		Make and investigate conjectures and provide examples and counter examples. *gg) Form opinions (conjectures) about a problem* *hh) Explore own and others' opinions about and within the context of a problem and be able to offer counter conjectures* *ii) Know how examples can be used to support an opinion in a problem* *jj) Use examples to support own and others' conjectures* *kk) Challenge ideas and conclusions with the support of examples*

Concept	Milestone	Learning
		When they have solved a problem, pose a similar problem for a peer. *ll) Identify the type of problem they have solved* *mm)Know the different elements of the types of problems* *nn) Recognise what makes a high-quality problem* *oo) Transfer skills and strategies from one problem/example to another*
Concept 3: Reasoning	1. *Provide a convinced argument with confidence and a chain of reasoning that makes sense to themselves (i.e. Using inductive reasoning)* 2. *Make use of a range of mathematical visuals (including tables to organise their process/findings/thoughts)*	*a) Understand which reasoning skills they are using when solving problems* *b) Begin to use their reasoning to select the skills and strategies required for problem solving* *c) To explain which reasoning skills they have used.* *d) To explain how those skills will help them solve the problem.* e) *To begin to reflect on how effectively they use their strategies.*
		Provide a convinced argument with confidence and a chain of reasoning that makes sense to themselves (i.e. Using inductive reasoning) *f) To be confident that their chain of reasoning is correct (even if the mathematical argument is not necessarily fully accurate)* *g) In the explanation use phrases such as 'I think…'* *h) To provide an explanation for what they did (i.e. their mathematical thought process) with reasons that are developing coherence and are mostly complete.*
		Make use of a range of mathematical visuals (including tables to organise their process/findings/thoughts) i) *To use a range of mathematical diagrams and/or visuals that are chosen specifically to support and aid the reasoning process and clearly demonstrate their understanding* j) *With support take a systematic approach* *k) To use mathematically accurate vocabulary in annotations and notes* *l) To ensure visuals, notes and calculations are organised and can be used to track their thought process (by themselves)*
Concept 4: Mathematical Communication	1. Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling. 2. *Evidence their mathematical thinking through clear verbal statements and written sentences* 3. *Edit and improve their own and a peer's convinced explanation.*	***Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.*** a) *To use precise mathematical vocabulary, with increasing accuracy and precision, to enhance their explanations.*
		Evidence their mathematical thinking through clear verbal statements and written sentences *b) To use complex verbal sentences to explain their thinking and mathematical processes with increasing accuracy* *c) To write down their thinking with the use of sentence stems, with increasing coherence and accuracy.* *d) To ask questions that challenge other children's explanations and/or thinking* *e) To ask questions that probe and challenge children's explanations* *f) To discuss the effectiveness of solutions with peers*
		Edit and improve their own and a peer's convinced explanation. *g) To improve their explanations based on feedback and self-reflection.* *h) To suggest how others might improve their explanations with clear feedback.*

MATHS Learning
Year 4

Concept	Milestone	Learning
Concept 1: Fluency	1. Become increasingly fluent with whole numbers and the four operations, including number facts and the concept of place value. 2. Develop efficient written and mental methods and perform calculations accurately with increasingly large whole numbers. 3. Develop their ability to solve a range of problems, including with simple fractions and decimal place value. 4. Draw with increasing accuracy and develop mathematical reasoning so they can analyse shapes and their properties, and confidently describe the relationships between them. 5. Use measuring instruments with accuracy and make connections between measure and number. 6. Know the multiplication tables up to and including the 12 multiplication table 7. Show precision and fluency in their work.	**Number & place value** a) count in multiples of 6, 7, 9, 25 and 1000 b) find 1000 more or less than a given number c) count backwards through zero to include negative numbers d) recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones) e) order and compare numbers beyond 1000 f) identify, represent and estimate numbers using different representations g) round any number to the nearest 10, 100 or 1000 h) solve number and practical problems that involve all of the above and with increasingly large positive numbers i) read Roman numerals to 100 (I to C) and know that over time, the numeral system changed to include the concept of zero and place value.
		Addition & subtraction j) add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate k) estimate and use inverse operations to check answers to a calculation l) solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.
		Multiplication & division m) recall multiplication and division facts for multiplication tables up to 12 × 12 n) use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers o) recognise and use factor pairs and commutativity in mental calculations p) multiply two-digit and three-digit numbers by a one-digit number using formal written layout q) solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.
		Fractions r) recognise and show, using diagrams, families of common equivalent fractions s) count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten. t) solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number u) add and subtract fractions with the same denominator v) recognise and write decimal equivalents of any number of tenths or hundredths

Concept	Milestone	Learning
		w) recognise and write decimal equivalents to $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$ x) find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths y) round decimals with one decimal place to the nearest whole number z) compare numbers with the same number of decimal places up to two decimal places aa) solve simple measure and money problems involving fractions and decimals to two decimal places.
		Measurement bb) Convert between different units of measure [for example, kilometre to metre; hour to minute] cc) measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres dd) find the area of rectilinear shapes by counting squares ee) estimate, compare and calculate different measures, including money in pounds and pence ff) read, write and convert time between analogue and digital 12- and 24-hour clocks gg) solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.
		Geometry – properties of shapes hh) compare and classify geometric shapes, including quadrilaterals and triangles**,** based on their properties and sizes ii) identify acute and obtuse angles and compare and order angles up to two right angles by size jj) identify lines of symmetry in 2-D shapes presented in different orientations kk) complete a simple symmetric figure with respect to a specific line of symmetry.
		Geometry – position & direction ll) describe positions on a 2-D grid as coordinates in the first quadrant mm) describe movements between positions as translations of a given unit to the left/right and up/down nn) plot specified points and draw sides to complete a given polygon.
		Statistics oo) interpret and present discrete and continuous data using appropriate graphical methods, including bar charts and time graphs. pp) solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and other graphs.
Concept 2: Problem Solving	1. *Make suggestions of ways to solve a range of problems.* 2. *Develop and apply a systematic approach.* 3. *Find and predict possibilities that match the context using patterns spotted to support.*	***Make suggestions of ways to solve a range of problems.*** a) Know and be able to identify the range/type of problems that can be solved (word problems, visual problems, finding all possibilities, logic, rules and patterns) b) Be able to identify the mathematical skill needed to solve the different types of problems (e.g. inverse, addition, multi-skill) c) Know (and use) the stages of problem solving (Nrich: Getting started, working on the problem, Digging Deeper, Reflecting)

Concept	Milestone	Learning
	4. *Independently check **and improve** work (e.g. look for other possibilities, repeats, missing answers, errors and **ways to improve**).* 5. *Begin to connect different but related patterns and use these to help solve problems.* 6. *Make and investigate conjectures and provide examples and counter- examples.* 7. *When they have solved a problem, pose a similar problem for a peer.*	*d)* d.Know the different ways that a problem can attempted to be solved (Nrich: Trial and Improvement, pattern spotting, working backwards, visualising) e) Choose a way to solve a problem from an agreed selection f) f. Using prior and current mathematical knowledge or experiences to make independent suggestions to solve a problem (I could…This is where I should start because…I have started this way because...If I do this first...)
		***Develop and apply a systematic approach**.* g) Know what it means to apply a systematic approach and the skills needed h) Know how a systematic approach ensures no possibilities are left out i) Use a systematic approach to ensure no possibilities are left out j) Know and explore how first guesses can be used to solve a problem k) Decide how to adapt an initial attempt or guess at a problem to try again l) Use information from initial thoughts and guesses to plan next steps m) Identify which information from an initial guess is useful (even if it is wrong) n) Plan steps taken to solve a problem o) *Make thought-out guesses about a problem and use these to inform next steps* p) *Refine or improve initial guesses and estimation*
		Find and predict possibilities that match the context using patterns spotted to support. q) Notice similarities and differences between examples r) Know what a mathematical pattern is s) Spot and describe patterns within a problem t) Recognise connections between possibilities and use these to find more u) Make predictions that relate to the context of a problem (e.g. I think the next number will be an odd number because 3 and 7 are both odd...) v) Generate new examples and solutions that fit the criteria of a problem based on patterns (e.g. the next numbers in the sequence could be 11 or 9)
		Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve). *w) Revisit solutions using learnt checking strategies* *x) Know what improvements can look like when solving a problem (adding in a missing number, omitting a repeat, spotting a better pattern or approach)* *y) Understand why improvements are important when problem solving* *z) Identify opportunities for improvements in their own and others' workings* *aa) Make improvements to workings that move them closer to a solution* bb) *Confirm they have found a correct solution by checking problems another way (working backwards, comparing with a partner, re-attempting the problem, explaining solutions aloud to a peer or adult, check-list, use a different representation (e.g. concrete instead of pictorial))*

Concept	Milestone	Learning
		Begin to connect different but related patterns and use these to help solve problems. *cc) Understand what connect means in a mathematical context* *dd) Demonstrate an awareness of how two (or more) patterns can be related* *ee) Begin to identify similarities and differences between two (or more patterns)* ff) *Use understanding of connected patterns (e.g....) to move closer to solutions*
		Make and investigate conjectures and provide examples and counter examples. *gg) Form opinions (conjectures) about a problem* *hh) Explore own and others' opinions about and within the context of a problem and be able to offer counter conjectures* *ii) Know how examples can be used to support an opinion in a problem* *jj) Use examples to support own and others' conjectures* kk) *Challenge ideas and conclusions with the support of examples*
		When they have solved a problem, pose a similar problem for a peer. *ll) Identify the type of problem they have solved* *mm) Know the different elements of the types of problems* *nn) Recognise what makes a high-quality problem* oo) *Transfer skills and strategies from one problem/example to another*
Concept 3: Reasoning	*1. Provide a convinced argument with confidence and a chain of reasoning that makes sense to themselves (i.e. Using inductive reasoning)* *2. Make use of a range of mathematical visuals (including tables to organise their process/findings/thoughts)*	*a) Understand which reasoning skills they are using when solving problems* *b) Begin to use their reasoning to select the skills and strategies required for problem solving* *c) To explain which reasoning skills they have used.* *d) To explain how those skills will help them solve the problem.* e) *To begin to reflect on how effectively they use their strategies.*
		Provide a convinced argument with confidence and a chain of reasoning that makes sense to themselves (i.e. Using inductive reasoning) *f) To be confident that their chain of reasoning is correct (even if the mathematical argument is not necessarily fully accurate)* *g) In the explanation use phrases such as 'I think…'* *h) To provide an explanation for what they did (i.e. their mathematical thought process) with reasons that are developing coherence and are mostly complete.*
		Make use of a range of mathematical visuals (including tables to organise their process/findings/thoughts) i) *To use a range of mathematical diagrams and/or visuals that are chosen specifically to support and aid the reasoning process and clearly demonstrate their understanding* j) *With support take a systematic approach* *k) To use mathematically accurate vocabulary in annotations and notes* *l) To ensure visuals, notes and calculations are organised and can be used to track their thought process (by themselves)*
Concept 4:	*1.* Pupils should read and spell mathematical vocabulary correctly and confidently, using	***Pupils should read and spell mathematical vocabulary correctly and confidently, using their growing word reading knowledge and their knowledge of spelling.***

Concept	Milestone	Learning
Mathematical Communication	their growing word reading knowledge and their knowledge of spelling. 2. *Evidence their mathematical thinking through clear verbal statements and written sentences* 3. *Edit and improve their own and a peer's convinced explanation.*	a) *To use precise mathematical vocabulary, with increasing accuracy and precision, to enhance their explanations.*
		Evidence their mathematical thinking through clear verbal statements and written sentences *b) To use complex verbal sentences to explain their thinking and mathematical processes with increasing accuracy* *c) To write down their thinking with the use of sentence stems, with increasing coherence and accuracy.* *d) To ask questions that challenge other children's explanations and/or thinking* *e) To ask questions that probe and challenge children's explanations* *f) To discuss the effectiveness of solutions with peers*
		Edit and improve their own and a peer's convinced explanation. *g) To improve their explanations based on feedback and self-reflection.* ***h)*** *To suggest how others might improve their explanations with clear feedback.*

MATHS Learning
Year 5

Concept	Milestone	Learning
Concept 1: Fluency	1. Extend their understanding of the number system and place value to include larger integers. 2. Develop the connections between multiplication and division with fractions, decimals, percentages and ratio. 3. Develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. 4. Begin to use the language of algebra as a means for solving a variety of problems. 5. Extend knowledge developed in number through the use of geometry and measures. 6. Classify shapes with increasingly complex geometric properties and learn the vocabulary needed to describe them. 7. Be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.	**Number & place value** 1. read, write, order and compare numbers to at least 1 000 000 and determine the value of each digit 2. count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 3. interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers, including through zero 4. round any number up to 1 000 000 to the nearest 10, 100, 1000, 10 000 and 100 000 5. solve number problems and practical problems that involve all of the above 6. read Roman numerals to 1000 (M) and recognise years written in Roman numerals. **Addition & subtraction** 7. add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction) 8. add and subtract numbers mentally with increasingly large numbers 9. use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy 10. solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why. **Multiplication & division** 11. identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers 12. know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers 13. establish whether a number up to 100 is prime and recall prime numbers up to 19 14. multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers 15. multiply and divide numbers mentally drawing upon known facts 16. divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context 17. multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 18. recognise and use square numbers and cube numbers, and the notation for squared (^{2}) and cubed (^{3}) 19. solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes 20. solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign 21. solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Concept	Milestone	Learning
		Fractions 22. compare and order fractions whose denominators are all multiples of the same number 23. identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths 24. recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, $\frac{2}{5} + \frac{4}{5} = \frac{6}{5} = 1\frac{1}{5}$] 25. add and subtract fractions with the same denominator and denominators that are multiples of the same number 26. multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams 27. read and write decimal numbers as fractions [for example, 0.71 = $\frac{71}{100}$] 28. recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents 29. round decimals with two decimal places to the nearest whole number and to one decimal place 30. read, write, order and compare numbers with up to three decimal places 31. solve problems involving number up to three decimal places 32. recognise the per cent symbol (%) and understand that per cent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal 33. solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.
		Measurement 34. convert between different units of metric measure (for example, kilometre and metre; centimetre and metre; centimetre and millimetre; gram and kilogram; litre and millilitre) 35. understand and use approximate equivalences between metric units and common imperial units such as inches, pounds and pints 36. measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres 37. calculate and compare the area of rectangles (including squares), and including using standard units, square centimetres (cm^2) and square metres (m^2) and estimate the area of irregular shapes 38. estimate volume [for example, using 1 cm^3 blocks to build cuboids (including cubes)] and capacity [for example, using water] 39. solve problems involving converting between units of time 40. use all four operations to solve problems involving measure [for example, length, mass, volume, money] using decimal notation, including scaling.
		Geometry – properties of shapes 41. identify 3-D shapes, including cubes and other cuboids, from 2-D representations 42. know angles are measured in degrees: estimate and compare acute, obtuse and reflex angles 43. draw given angles, and measure them in degrees (o) 44. identify: 1. angles at a point and one whole turn (total 360o)

Concept	Milestone	Learning
		2. angles at a point on a straight line and $\frac{1}{2}$ a turn (total 180o) 3. other multiples of 90o 45. use the properties of rectangles to deduce related facts and find missing lengths and angles 46. distinguish between regular and irregular polygons based on reasoning about equal sides and angles.
		Geometry – position & direction 47. identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.
		Statistics 48. solve comparison, sum and difference problems using information presented in a line graph 49. complete, read and interpret information in tables, including timetables.
Concept 2: Problem Solving	*1. Organise work from the outset, looking for ways to record and work systematically.* *2. Identify and connect different but related patterns and use these to help solve problems.*	***Organise work from the outset, looking for ways to record and work systematically.*** a) Know and use a range of ways to record and organise thinking when solving a problem (jottings in the moment, drawings, photos, tables, lists, use of keys) b) Use ways to record and work that visualise their thinking for themselves and others c) Choose efficient and clear ways to record d) Consider steps needed to solve a problem before starting e) Follow a pattern or system when exploring a problem f) Consciously choose a way to solve a problem that shows others they are using a pattern/system
		Identify and connect different but related patterns and use these to help solve problems. *g) Seek out patterns within a range of problems* *h) Make comparisons to draw relationships and connections between identified patterns* *i) Understand and apply what makes two or more patterns connected* j) *Determine and filter which related patterns will be effective to help solve problems and use these to reach conclusions*
Concept 3: Reasoning	*1. Provide a clear, correct, logical justification with a complete chain of mathematically credible reasoning - Year 5* *2. Provide proof of reasoning, using a watertight argument that is mathematically sound, based on generalisations and underlying mathematical structure (i.e. deductive reasoning) – Year 6* *3. Express generalisations in words and symbolic notation, including through the use of algebra*	*a) Use their reasoning skills to make conscious choices about which problem solving skills and approaches are most effective for solving a problem.* *b) To make very conscious and deliberate choices about the skills they are using.* *c) To explain what the impact and benefit is of those skills in relation to the problem being solved.* d) *To know which strategies they use effectively and which they find more challenging or need to develop further.*
		Provide a clear, correct, logical justification with a complete chain of mathematically credible reasoning (Year 5) *e) To provide a correct logical argument that has a complete chain of reasoning* *f) In the explanation use words such as 'because', 'therefore', 'and so', 'that leads to' ...*
		Make accurate use of deliberately chosen mathematical visuals (e.g. diagrams, tables, graphs) g) *To accurately use a range of mathematical diagrams, tables, graphs to capture and crystallise the reasoning process and clearly demonstrate their understanding* h) *Independently make deliberate choices so they take a systematic approach* *i) To use precise mathematical vocabulary in all annotations and notes*

Concept	Milestone	Learning
	4. Make accurate use of deliberately chosen mathematical visuals (e.g. diagrams, tables, graphs)	*j) To ensure visuals, notes and calculations are well organised and are used to accurately articulate their thought process (by themselves and others)*
		Express generalisations in words and symbolic notation, including through the use of algebra
		k) To understand algebraic concepts and notation to help them express their reasoning in a succinct and mathematically elegant manner.
Concept 4: Mathematical Communication	1. Pupils should read, spell and pronounce mathematical vocabulary correctly.	***Pupils should read, spell and pronounce mathematical vocabulary correctly.***
		a) *To consistently use precise mathematical vocabulary, with accuracy, in explanations.*
	2. Be able to communicate their reasoning in writing without any verbal explanation/interpretation	***Be able to communicate their reasoning in writing without any verbal explanation/interpretation***
		b) To give clear, accurate verbal explanations that are increasingly sophisticated.
		c) To write complex sentences independently that explain their thinking clearly to others without verbal explanation.
	3. Reflect on others' justification/ proof and use this to improve their own work	*d) To decide what information to record and when, based on its mathematical relevance and/or significance.*
		Reflect on others' justification/ proof and use this to improve their own work
	4. Edit and improve their own and a peer's justification – Year 5	*e) To give feedback to other people about their explanations, giving reasons for their feedback that are based on their knowledge of effective reasoning and mathematical communication.*
	5. Edit and improve their own and a peer's proof – Year 6	***Edit and improve their own and a peer's justification (Year 5)***
		f) To make precise refinements of their explanations based primarily on self-reflection, but also feedback from others.
	6. To ask questions of other children to help move their thinking forwards	*g) To give others increasingly insightful and specific feedback about how to refine their explanations.*

MATHS Learning
Year 6

Concept	Milestone	Learning
Concept 1: Fluency	1. Extend their understanding of the number system and place value to include larger integers. 2. Develop the connections between multiplication and division with fractions, decimals, percentages and ratio. 3. Develop their ability to solve a wider range of problems, including increasingly complex properties of numbers and arithmetic, and problems demanding efficient written and mental methods of calculation. 4. Begin to use the language of algebra as a means for solving a variety of problems. 5. Extend knowledge developed in number through the use of geometry and measures. 6. Classify shapes with increasingly complex geometric properties and learn the vocabulary needed to describe them. 7. Be fluent in written methods for all four operations, including long multiplication and division, and in working with fractions, decimals and percentages.	**Number & place value** a) read, write, order and compare numbers up to 10 000 000 and determine the value of each digit b) round any whole number to a required degree of accuracy c) use negative numbers in context, and calculate intervals across zero d) solve number and practical problems that involve all of the above. **Addition & subtraction** e) multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication f) divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context g) divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context h) perform mental calculations, including with mixed operations and large numbers i) identify common factors, common multiples and prime numbers j) use their knowledge of the order of operations to carry out calculations involving the four operations k) solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why **Multiplication & division** l) solve problems involving addition, subtraction, multiplication and division m) use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy. **Fractions** n) use common factors to simplify fractions; use common multiples to express fractions in the same denomination o) compare and order fractions, including fractions > 1 p) add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions q) multiply simple pairs of proper fractions, writing the answer in its simplest form [for example, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$] r) divide proper fractions by whole numbers [for example, $\frac{1}{3} \div 2 = \frac{1}{6}$]

Concept	Milestone	Learning
		s) associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction [for example, $\frac{3}{8}$] t) identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places u) multiply one-digit numbers with up to two decimal places by whole numbers v) use written division methods in cases where the answer has up to two decimal places w) solve problems which require answers to be rounded to specified degrees of accuracy x) recall and use equivalences between simple fractions, decimals and percentages, including in different contexts.
		Ratio & proportion y) solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts z) solve problems involving the calculation of percentages [for example, of measures, and such as 15% of 360] and the use of percentages for comparison aa) solve problems involving similar shapes where the scale factor is known or can be found bb) solve problems involving unequal sharing and grouping using knowledge of fractions and multiples.
		Algebra cc) use simple formulae dd) generate and describe linear number sequences ee) express missing number problems algebraically ff) find pairs of numbers that satisfy an equation with two unknowns gg) enumerate possibilities of combinations of two variables.
		Measurement hh) solve problems involving the calculation and conversion of units of measure, using decimal notation up to three decimal places where appropriate ii) use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to up to three decimal places jj) convert between miles and kilometres kk) recognise that shapes with the same areas can have different perimeters and vice versa ll) recognise when it is possible to use formulae for area and volume of shapes mm) calculate the area of parallelograms and triangles nn) calculate, estimate and compare volume of cubes and cuboids using standard units, including cubic centimetres (cm3) and cubic metres (m3), and extending to other units [for example, mm3 and km3].

Concept	Milestone	Learning
		Geometry – properties of shapes oo) draw 2-D shapes using given dimensions and angles pp) recognise, describe and build simple 3-D shapes, including making nets qq) compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons rr) illustrate and name parts of circles, including radius, diameter and circumference and know that the diameter is twice the radius ss) recognise angles where they meet at a point, are on a straight line, or are vertically opposite, and find missing angles.
		Geometry – position & direction tt) describe positions on the full coordinate grid (all four quadrants) uu) draw and translate simple shapes on the coordinate plane, and reflect them in the axes.
		Statistics vv) interpret and construct pie charts and line graphs and use these to solve problems ww) calculate and interpret the mean as an average.
Concept 2: Problem Solving	*1. Organise work from the outset, looking for ways to record and work systematically.* *2. Identify and connect different but related patterns and use these to help solve problems.*	***Organise work from the outset, looking for ways to record and work systematically.*** a) Know and use a range of ways to record and organise thinking when solving a problem (jottings in the moment, drawings, photos, tables, lists, use of keys) b) Use ways to record and work that visualise their thinking for themselves and others c) Choose efficient and clear ways to record d) Consider steps needed to solve a problem before starting e) Follow a pattern or system when exploring a problem f) Consciously choose a way to solve a problem that shows others they are using a pattern/system
		Identify and connect different but related patterns and use these to help solve problems. *g) Seek out patterns within a range of problems* *h) Make comparisons to draw relationships and connections between identified patterns* *i) Understand and apply what makes two or more patterns connected* j) *Determine and filter which related patterns will be effective to help solve problems and use these to reach conclusions*
Concept 3: Reasoning	*1. Provide a clear, correct, logical justification with a complete chain of mathematically credible reasoning - Year 5* *2. Provide proof of reasoning, using a watertight argument that is mathematically sound, based on generalisations and underlying mathematical structure (i.e. deductive reasoning) – Year 6*	*a) Use their reasoning skills to make conscious choices about which problem solving skills and approaches are most effective for solving a problem.* *b) To make very conscious and deliberate choices about the skills they are using.* *c) To explain what the impact and benefit is of those skills in relation to the problem being solved.* d) *To know which strategies they use effectively and which they find more challenging or need to develop further.*
		Provide proof of reasoning, using a watertight argument that is mathematically sound, based on generalisations and underlying mathematical structure (i.e. deductive reasoning) – Year 6

Concept	Milestone	Learning
	3. Express generalisations in words and symbolic notation, including through the use of algebra *4. Make accurate use of deliberately chosen mathematical visuals (e.g. diagrams, tables, graphs)*	e) *To provide a watertight argument that is mathematically sound, often based on generalisations and underlying structure.*
		Make accurate use of deliberately chosen mathematical visuals (e.g. diagrams, tables, graphs) f) *To accurately use a range of mathematical diagrams, tables, graphs to capture and crystallise the reasoning process and clearly demonstrate their understanding* g) *Independently make deliberate choices so they take a systematic approach* *h) To use precise mathematical vocabulary in all annotations and notes* i) *To ensure visuals, notes and calculations are well organised and are used to accurately articulate their thought process (by themselves and others)*
		Express generalisations in words and symbolic notation, including through the use of algebra j) *To understand algebraic concepts and notation to help them express their reasoning in a succinct and mathematically elegant manner.*
Concept 4: Mathematical Communication	1. Pupils should read, spell and pronounce mathematical vocabulary correctly. *2. Be able to communicate their reasoning in writing without any verbal explanation/interpretation* *3. Reflect on others' justification/ proof and use this to improve their own work* *4. Edit and improve their own and a peer's justification – Year 5* *5. Edit and improve their own and a peer's proof – Year 6* *6. To ask questions of other children to help move their thinking forwards*	***Pupils should read, spell and pronounce mathematical vocabulary correctly.*** a) *To consistently use precise mathematical vocabulary, with accuracy, in explanations.*
		Be able to communicate their reasoning in writing without any verbal explanation/interpretation *b) To give clear, accurate verbal explanations that are increasingly sophisticated.* *c) To write complex sentences independently that explain their thinking clearly to others without verbal explanation.* d) *To decide what information to record and when, based on its mathematical relevance and/or significance.*
		Reflect on others' justification/ proof and use this to improve their own work e) *To give feedback to other people about their explanations, giving reasons for their feedback that are based on their knowledge of effective reasoning and mathematical communication.*
		Edit and improve their own and a peer's proof – Year 6 *f) To make precise refinements of their explanations based primarily on self-reflection, but also feedback from others.* g) *To give others increasingly insightful and specific feedback about how to refine their explanations.*